## Pairwise Sequence Alignment (II)

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## Pairwise Alignment Algorithm Using Dynamic Programming

- Initialization: Given two sequences with length $m$ and $n$, create a $(\mathrm{m}+1) \times(\mathrm{n}+1)$ matrix M. Initialize the first row and first column according to scoring matrix.
- For j in 1 ..n (column)
for i in $1 . . \mathrm{m}$ (row)
$M[i, j]=\max ((M[i-1, j-1]+S(i, j), M[i, j-1]+S(-$ ,j), M[i-1, j] + S(i,-) )
Record the selected path toward ( $\mathrm{i}, \mathrm{j}$ )
- Report alignment score $M[m][n]$ and trace back to $M[0,0]$ to generate the optimal alignment.


## Local Sequence Alignment Using DP

- Biological sequences usually only have local similarity. For instance: a protein sequence may consist of a few modules. Two proteins may only have one similar modules, whereas other regions are not similar at all.
- During evolution, only functional and structural important regions are highly conserved.
- Global alignment sacrifices the local similarity to maximize the global alignment score.
- We need to use alignment method to identify the local similar regions disregard of other dissimilar regions.


## Local vs. Global Alignment

- Global Alignment

- Local Alignment—better alignment to find conserved segment
tccCAGTTATGTCAGgggacacgagcatgcagagac |||||||||||
aattgccgccgtcgttttcagCAGTTATGTCAGatc


## Local Alignment Algorithm

Goal: find an alignment of the substrings of P and Q with maximum alignment score.
Naïve Algorithm:
$(\mathrm{m}+1) * \mathrm{~m} / 2$ substrings of $\mathrm{P},(\mathrm{n}+1) * \mathrm{n} / 2$ substrings of Q Using DP for each substring pairs:
$\mathrm{m}^{\wedge} 2 * \mathrm{n}^{\wedge} 2 * \mathrm{O}(\mathrm{mn})=\mathrm{O}\left(\mathrm{m}^{\wedge} 3 \mathrm{n}^{\wedge} 3\right)$
(too slow!)

## Smith-Waterman Algorithm

Same Dynamic Program algorithm as global alignment except for three differences.

1. All negative scores is converted to 0 (why?)
2. Alignment can start from anywhere in the matrix
3. Alignment can end at anywhere in the matrix

## Local Alignment Algorithm

- Initialization: Given two sequences with length $m$ and $n$, create a $(\mathrm{m}+1) \times(\mathrm{n}+1)$ matrix M. Initialize the first row and first column to 0 s .
- For j in 1..n (column)

$$
\text { for } \mathrm{i} \text { in } 1 . . \mathrm{m} \text { (row) }
$$

$$
\mathrm{M}[\mathrm{i}, \mathrm{j}]=\max (0,(\mathrm{M}[\mathrm{i}-1, \mathrm{j}-1]+\mathrm{S}(\mathrm{i}, \mathrm{j}), \mathrm{M}[\mathrm{i}, \mathrm{j}-
$$

$$
1]+S(-, j), M[i-1, j]+S(i,-))
$$

Record the selected path.

- Find elements in matrix M with maximum values. Trace back till 0 and report the alignment corresponding to the path.

1. Initialization

|  |  | A |  |  | A |  | G |  | A |  |  | T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | 0 | 0 | 0 |  | 0 |  | 0 | 0 |  | 0 |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| G | 0 |  |  |  |  |  |  |  |  |  |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| T | 0 |  |  |  |  |  |  |  |  |  |  |  |
| C | 0 |  |  |  |  |  |  |  |  |  |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| G | 0 |  |  |  |  |  |  |  |  |  |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| A | 0 |  |  |  |  |  |  |  |  |  |  |  |
| T | 0 |  |  |  |  |  |  |  |  |  |  |  |
| G | 0 |  |  |  |  |  |  |  |  |  |  |  |

3. Trace Back

4. Trace Back


# Two Best Local Alignments 

Local Alignment 1:
ATCAGAA
AT-AGAA

Local Alignment 2:
ATCAGAAAT
AT-AGA-AT

## Affine Gap Penalty

- We have treated gaps independently. However, gaps in biological sequences are dependent.
- Hard to open the first gap, and easy to extend gaps.
- First gap should be penalized more than extended gaps.


## Affine Gap Function

- Penalty of gaps $=\mathrm{Wo}+\mathrm{We} *(\mathrm{~g}-1)$
- Wo: penalty of opening a gap
- We: penalty of extending a gap
- g: gap size
- Example: $\mathrm{Wo}=-2, \mathrm{We}=-1$


## Example:

$$
\begin{aligned}
& \text { AGATCAGAAATG } \\
& \text {--AT-AG-AAT- }
\end{aligned}
$$

Alignment score: $-2-1+1+1-2+1+1-2+1+1+1-2=-2$

## Complexity of Affine Gap Penalty

- To compute the alignment score, we need to know the state of the previous alignment: last pair matches two characters? last pair matches $\mathrm{P}[\mathrm{k}]$ with a gap? last pair matches $\mathrm{Q}[\mathrm{t}]$ with a gap?
So we need to introduce more matrices to record the scores in these three situations.


## Dynamic programming with Affine Gap Penalty

- Algorithms proceeds by aligning $\mathrm{P}[1 . . \mathrm{i}]$ with $\mathrm{Q}[1 . . \mathrm{j}]$. For these prefixes of P and Q , define the following four matrices V, G, F, E:
- $\mathrm{V}[i, j]$ is the value of an optimal alignment of prefix $\mathrm{P}[1 . . \mathrm{i}]$ and prefix $\mathrm{Q}[1 . . j]$.
- $\mathrm{G}[i, j]$ is the value of an optimal alignment of $\mathrm{P}[1 . . \mathrm{i}]$ and $\mathrm{Q}[1 . . \mathrm{j}]$ whose last pair matches $\mathrm{P}[\mathrm{i}]$ with $\mathrm{Q}[\mathrm{j}]$
- $F(i, j)$ is the value of an optimal alignment of $P[1 . . i]$ and $\mathrm{T}[1 . . \mathrm{j}]$ whose last pair matches $\mathrm{P}[\mathrm{i}]$ with a gap.
- $E(i, j)$ is the value of an optimal alignment of $P[1 . . i]$ and $\mathrm{Q}[1 . \mathrm{j}]$ whose last pair matches a gap with $\mathrm{Q}[\mathrm{j}]$


## Initialization

- $\mathrm{V}(0,0)=0$
- $\mathrm{V}(\mathrm{i}, 0)=\mathrm{Wo}+\mathrm{We} *(\mathrm{i}-1)$
- $\mathrm{V}(0, \mathrm{j})=\mathrm{Wo}+\mathrm{We}$ *(j-1)
- $\mathrm{E}(\mathrm{i}, 0)=-\infty$
- $F(0, j)=-\infty$


## Fill Matrix

- $\mathrm{V}(\mathrm{i}, \mathrm{j})=\max (\mathrm{G}(\mathrm{i}, \mathrm{j}), \mathrm{F}(\mathrm{i}, \mathrm{j}), \mathrm{E}(\mathrm{i}, \mathrm{j}))$
- $\mathrm{G}(\mathrm{i}, \mathrm{j})=\mathrm{V}(\mathrm{i}-1, \mathrm{j}-1)+\mathrm{S}(\mathrm{P}[\mathrm{i}], \mathrm{Q}[\mathrm{j}])$
- $F(i, j)=\max (F(i-1, j)+W e, G(i-1, j)+W o$, $E(i-1, j)+W o$
- $\mathrm{E}(\mathrm{i}, \mathrm{j})=\max (\mathrm{E}(\mathrm{i}, \mathrm{j}-1)+\mathrm{We}, \mathrm{G}(\mathrm{i}, \mathrm{j}-1)+$ Wo, $\mathrm{F}(\mathrm{i}, \mathrm{j}-1)+\mathrm{Wo})$


## Trace Back

- Trace within matrix and across matrices
- Time and space complexity: $\mathrm{O}(\mathrm{m} * \mathrm{n})$
- Comments: four matrices. 3 matrices for three different possible ending states. 1 matrix is the master matrix to record the best alignment scores from those three matrices.


## Comments

- Does sequence alignment based on mathematical optimization really make biological sense?
- When alignment is good?
- When alignment is bad?
- Alignment quality and sequence identity relationship


# Application Example (Alignment - Structure) 

TARGET

ASILPKRLFGNCEQTSDEGLK IERTPLVPHISAQNVCLKIDD VPERLIPERASFQWMNDK

TEMPLATE



ASILPKRLFGNCEQTSDEGLKIERTPLVPHISAQNVCLKIDDVPERLIPE MSVIPKRLYGNCEQTSEEAIRIEDSPIV---TADLVCLKIDEIPERLVGE


Source: A. Fisher, 2005

## Sequence Identity and Alignment Quality in Structure Prediction




Superimpose
-> RMSD
\%Sequence Identity: percent of identical residues in alignment RMSD: square root of average distance between predicted structure and native structure.

## Global and Local Alignment Tools

- NEEDLE (global alignment)
http://bioweb.pasteur.fr/seqanal/interfaces/needle. $\underline{\mathrm{html}}$
- WATER (local alignment)
http://bioweb.pasteur.fr/seqanal/interfaces/water.ht ml

