# Computational Gene Finding 



## Dong Xu

Digital Biology Laboratory
Computer Science Department Christopher S. Life Sciences Center University of Missouri, Columbia E-mail: xudong@missouri.edu http://digbio.missouri.edu

## Lecture Outline

- Protein-encoding genes and gene structures
- Computational models for coding regions
- Computational models for coding-region boundaries
- Markov chain model for coding regions


## What Is a Gene?

Definition: A gene is the nucleotide sequence that stores the information which specifies the order of the monomers in a final functional polypeptide or RNA molecule, or set of closely related isoforms (Epp CD, Nature, 389: 537).

## Gene and Disease

## Environment

## Genes

## Monogenic Diseases

- Cystic fibrosis
- Huntington' s disease
- Haemophilia
- Phenylketonuria


## Common Diseases

- Alzheimer disease
- Adult onset diabetes
- Cancer
- Cardiovascular disease
- Depression


## Infections

- Influenza
- Hepatitis
- AIDS


## Genetic Code



## Reading Frame

- Reading (or translation) frame: each DNA segment has six possible reading frames


## Forward strand:

Reading frame \#1
ATG
GCT
TAC
GCT TGC

## Reverse strand:

Reading frame \#4
TCA

AGC
GTA
AGC
CAT

## ATGGCTACGCTTGA

| Reading frame \#2 | Reading |
| :--- | :--- |
| TGG | GGC |
| CTT | TTA |
| ACG | CGC |
| CTT | TTG |
| GA. | A.. |

## TCAAGCGTAAGCCAT

| Reading frame \#5 | Reading |
| :--- | :--- |
| CAA | AAG |
| GCG | CGT |
| TAA | AAG |
| GCC | CCA |
| AT. | T.. |

## Prokaryotic Gene Structure

$\square$

Coding region of Open Reading Frame
Promoter region (maybe)

Ribosome binding site (maybe)

Termination sequence (maybe)

Start codon / Stop Codon


Open reading frame (ORF): a segment of DNA with two in-frame stop codons at the two ends and no in-frame stop codon in the middle

## Eukaryotic Gene Structure



## Gene Structure Rules

- Each coding region (exon) has a fixed translation frame (no gaps allowed)
- All exons of a gene are on the same strand
- Neighboring exons of a gene can have different reading frames



## Computational Gene

 Finding- The Problem: Given a stretch of DNA sequence, find all coding regions and construct gene structures from identified exons if needed atgaacagacgcgatcttcttttacaagaaatgggcatttcccagtgggaattatatcgc cccqaggtactgcaaggttcaqtagyaat tagtytggcagagaatattcgccttaptcact gtttccgatgaaaatatcagtagctcgcctttgttggctgatgtgctgttaagccttaat cttaalaaagaaaattgtttatgtttgaattacgatcaaatccagcatatggaatgtaaa \&agcctattcgttattggttactatcagaaaatagcgaccaaattbaccgcactttgcca. ttttgcaagcaggctgagcaggtttatcgctcgccaagttggcagcaatttcaatctaat catceaqccaaacqaqcottqtqqcaacaalttcaqcaqccttaa
- A gene finding problem can be decomposed into two problems:
$K$ identification of coding potential of a region in a particular frame
$K$ identification of boundaries between coding and non-coding regions


## Repetitive Sequence

- Definition

KDNA sequences that made up of copies of the same or nearly the same nucleotide sequence
$K$ Present in many copies per chromosome set

## Repeat Filtering

## - RepeatMasker

$\measuredangle$ Uses precompiled representative sequence libraries to find homologous copies of known repeat families
$K$ Use Blast
Khttp://www.repeatmasker.org/

## Gene Finding Tools

- Genscan
(http://genes.mit.edu/GENSCAN.html )
- GeneMarkHMM
(http://opal.biology.gatech.edu/GeneMark/)
- GRAIL (http://compbio.ornl.gov/Grail-1.3/)
- Genie
(http://www.fruitfly.org/seq tools/genie.html)
- Glimmer
(http://www.tigr.org/softlab/glimmer)


## Testing Finding Tools

- Access Genscan (http://genes.mit.edu/GENSCAN.html )
- Use a sequence at
http://www.ncbi.nlm.nih.gov/entrez/viewer.fcgi?db=nucleotide\&val=8077108


## Lecture Outline

- Protein-encoding genes and gene structures
- Computational models for coding regions
- Computational models for coding-region boundaries
- Markov chain model for coding regions


## Coding Signal Detection (1)

- Frequency distribution of dimers in protein sequence (shewanella)

| Name | ala | arg | asn | asp | cys | gu | gn | gly | his | Ile | leu | lys | met | he | pro |  |  | rp |  | val |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ala | 9.5 | 4.1 | 4.3 | 5.3 | 1.2 | 6 | 4.8 | 6.5 | 2 | 6.5 | 11.5 | 6 | 2.6 | 3.7 | 3.5 | 6.2 | 5 | 1 | 2.7 | 6.5 |
| arg | 7.9 | 5.5 | 3.9 | 5.3 | 1.1 | 6 | 5.5 | 5.9 | 2.6 | 6.5 | 11.4 | 5 | 2.2 | 4.7 | 3.6 | 5.5 | 4.4 | 1.4 | 4 | 6.6 |
| asn | 9.6 | 4.9 | 4.2 | 4.9 | 1 | 5.3 | 5.6 | 7.4 | 2.3 | 6 | 10 | 4.9 | 2 | 3.5 | 5.1 | 1 | 5.5 | 1.5 | . 1 | 6.1 |
| as | 9.3 | 4 | 4.7 | 5.1 | 1 | 6.7 | 2.9 | 7 | 1.8 | 7.1 | 9.6 | 6.3 | 2.3 | 4.3 | 3.9 | 5.9 | 5.1 | 1.6 | 3.6 | 6.6 |
| cys | 8.4 | 4.8 | 3.3 | 5.4 | 1.7 | 5.6 | 5.2 | 8.1 | 4.3 | 5.4 | 10.2 | 3.8 | 1.8 | 4.1 | 4.5 | 6.3 | 4.3 | 1.6 | 3.4 | 6.8 |
| glu | 9.4 | 5.8 | 3.6 | 4.5 | 0.8 | 4.9 | 7 | 5.8 | 2.6 | 5.9 | 12.7 | 5 | 2.4 | 4 | 3.5 | 5.4 | 5 | 1.1 | 2.8 | 6.8 |
| gln | 10.3 | 4.9 | 3 | 4.4 | 0.9 | 4.5 | 6.8 | 7 | 2.7 | 5.5 | 12.8 | 4.1 | 2 | 3.9 | 3.8 | 5.8 | 5.3 | 1.4 | 3 | 6.9 |
| gly | 8.1 | 4.8 | 3.9 | 5.1 | 1.2 | 6 | 4.6 | 6.4 | 2.4 | 6.8 | 10.5 | 5.8 | 2.7 | 4.8 | 2.4 | 5.8 | 5.1 | 1.4 | 3.7 | 7.5 |
| his | 7.3 | 4.7 | 4 | 4.8 | 1.5 | 4.9 | 5.6 | 6.9 | 3 | 6.2 | 10.8 | 4.8 | 1.6 | 5 | 5.2 | 6.8 | 4.9 | 1.7 | 4.2 | 5.1 |
| ile | 11 | 4.7 | 4.9 | 6.5 | 1.1 | 6.9 | 3.6 | 7.2 | 2.1 | 5.3 | 8.6 | 5.3 | 1.8 | 3.2 | 4.2 | 7 | 5.6 | 0.9 | 2.9 | 6.1 |
| leu | 10.4 | 4.2 | 4.3 | 5.2 | 1.1 | 5.2 | 3.7 | 6.8 | 2 | 5.6 | 10.6 | 5.3 | 2.3 | 3.8 | 4.5 | 7.4 | 6.2 | 1 | 2.6 | 6.6 |
| lys | 10.6 | 5.2 | 3.8 | 5.2 | 0.5 | 5.3 | 5.9 | 6.6 | 2.6 | 5.2 | 11.3 | 4.7 | 1.9 | 2.8 | 4.6 | 6 | 5.5 | 1.2 | 2.6 | 7.6 |
| met | 10.8 | 4.8 | 3.8 | 4.6 | 0.7 | 4.6 | 4.9 | 7 | 1.7 | 4.7 | 11.4 | 5.2 | 2.8 | 3.3 | 5.1 | 7.4 | 6.3 | 0.9 | 2 | 6.8 |
| phe | 9.6 | 3.7 | 5.2 | 6.5 | 1.2 | 6.4 | 2.7 | 7.9 | 1.9 | 6.7 | 7.4 | 5 | 2.5 | 3.9 | 3.6 | 8 | 5.8 | 1.3 | 3.3 | 6.3 |
| pro | 8.4 | 3.6 | 4.6 | 5.4 | 0. | 7.6 | 5.2 | 5.4 | 2.3 | 6.1 | 11.2 | 5.5 | 2.4 | 4.2 | 2.8 | 6.5 | 5.4 | 1.4 | 2.9 | 7.5 |
| ser | 9.1 | 4.6 | 3.7 | 5 | 1 | 5.4 | 5.2 | 7.2 | 2.6 | 6 | 11.6 | 4.5 | 2.2 | 4.1 | 4.1 | 6.5 | 5 | 1.2 | 3.2 | 6.8 |
| thr | 9.1 | 4.2 | 3.7 | 5.6 | 0.9 | 5.7 | 5.7 | 7.5 | 2.2 | 5.5 | 12 | 4.2 | 2 | 3.5 | 5.5 | 6.2 | 5.3 | 1.1 | 2.6 | 6.7 |
| trp | 7.1 | 6.3 | 3.2 | 4.8 | 1.3 | 3.9 | 8.5 | 6.6 | 3.6 | 5 | 14.2 | 3.2 | 2.4 | 4.6 | 3.9 | 5.8 | 4.3 | 1.3 | 3 | 6.1 |
| tyr | 7.9 | 6.5 | 3.6 | 4.9 | 1.2 | 4.5 | 7 | 7.1 | 2.6 | 5 | 11.7 | 4 | 1.6 | 4.7 | 4.9 | 6.4 | 4.6 | 1.5 | 3.4 | 5.7 |
| val | 9.6 | 4.1 | 4.4 | 5.9 | 1 | 6.2 | 3.4 | 6.4 | 1.8 | 6.5 | 10.2 | 5.2 | 2.5 | 3.7 | 3.8 | 7.2 | 6.1 | 1.1 | 2.7 | 7.1 |

## The average frequency is $5 \%$

## Some amino acids prefer to be next to each other

Some other amino acids prefer to be not next to each other

## Coding Signal Detection (2)

- Dimer bias (or preference) could imply di-codon (6-mers like AAA TTT) bias in coding versus non-coding regions
- Relative frequencies of a di-codon in coding versus non-coding
$K$ frequency of dicodon $X$ (e.g, AAAAAA) in coding region, total number of occurrences of $X$ divided by total number of dicocon occurrences
$K$ frequency of dicodon $X$ (e.g, AAAAAA) in noncoding region, total number of occurrences of $X$ divided by total number of dicodon occurrences

In human genome, frequency of dicodon "AAA AAA" is
$\sim 1 \%$ in coding region versus $\sim 5 \%$ in non-coding region
Question: if you see a region with many "AAA AAA", would you guess it is a coding or non-coding region?

## Coding Signal Detection (3)

- Most dicodons show bias towards either coding or noncoding regions; only fraction of dicodons is neutral
- Foundation for coding region identification

Regions consisting of dicodons that mostly tend to be in coding regions are probably coding regions; otherwise non-coding regions

- Dicodon frequencies are key signal used for coding region detection; all gene finding programs use this information


## Coding Signal Detection (4)

## - Dicodon frequencies in coding versus non-coding are genome-dependent

| Name | ala |  |  |  |  |  |  |  |  | ile | leu |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ala | 9.5 | 4.1 | 4.3 | 5.3 | 1.2 | . 6 | 4.8 | 6.5 | 2 | 6.5 | 11.5 | 6 | 2.6 | 3.7 | 3.5 | 6.2 | 5 | 1.1 | 2.7 | . 5 |
| arg | 7.9 | 5.5 | 3.9 | 5.3 | 1.1 | - 6 | 5.5 | 5.9 | 2.6 | 6.5 | 11.4 | 5 | 2.2 | 4.7 | 3.6 | 5.5 | 4.4 | 1.4 | 4 | . 6 |
| asn | 9.6 | 4.9 | 4.2 | 4.9 | 1 | 5.3 | 5.6 | 7.4 | 2.3 | 6 | 10 | 4.9 | 2 | 3.5 | 5.1 | 6.1 | 5.5 | 1.5 | 3.1 | 6.1 |
| asp | 9.3 | 4 | 4.7 | 5.1 | 1 | 6.7 | 2.9 | 7 | 1.8 | 7.1 | 9.6 | 6.3 | 2.3 | 4.3 | 3.9 | 5.9 | 5.1 | 1.6 | 3.6 | 6.6 |
| cys | 8.4 | 4.8 | 3.3 | 5.4 | 1.7 | 5.6 | 5.2 | 8.1 | 4.3 | 5.4 | 10.2 | 3.8 | 1.8 | 4.1 | 4.5 | 6.3 | 4.3 | 1.6 | 3.4 | 6.8 |
| glu | 9.4 | 5.8 | 3.6 | 4.5 | 0.8 | 4.9 | 7 | 5.8 | 2.6 | 5.9 | 12.7 | 5 | 2.4 | 4 | 3.5 | 5.4 | 5 | 1.1 | 2.8 | 6.8 |
| gln | 10.3 | 4.9 | 3 | 4.4 | 0.9 | 4.5 | 6.8 | 7 | 2.7 | 5.5 | 12.8 | 4.1 | 2 | 3.9 | 3.8 | 5.8 | 5.3 | 1.4 | 3 | 6.9 |
| gly | 8.1 | 4.8 | 3.9 | 5.1 | 1.2 | . 6 | 4.6 | 6.4 | 2.4 | 6.8 | 10.5 | 5.8 | 2.7 | 4.8 | 2.4 | 5.8 | 5.1 | 1.4 | 3.7 | 7.5 |
| his | 7.3 | 4.7 | 4 | 4.8 | 1.5 | 4.9 | 5.6 | 6.9 | 3 | 6.2 | 10.8 | 4.8 | 1.6 | 5 | 5.2 | 6.8 | 4.9 | 1.7 | 4.2 |  |
| ile | 11 | 4.7 | 4.9 | 6.5 | 1.1 | 6.9 | 3.6 | 7.2 | 2.1 | 5.3 | 8.6 | 5.3 | 1.8 | 3.2 | 4.2 | 7 | 5.6 | 0.9 | 2.9 | 6.1 |
| leu | 10.4 | 4.2 | 4.3 | 5.2 | 1.1 | 5.2 | 3.7 | 6.8 | 2 | 5.6 | 10.6 | 5.3 | 2.3 | 3.8 | 4.5 | 7.4 | 6.2 | 1 | 2.6 |  |
| lys | 10.6 | 5.2 | 3.8 | 5.2 | 0.5 | 5.3 | 5.9 | 6.6 | 2.6 | 5.2 | 11.3 | 4.7 | 1.9 | 2.8 | 4.6 | 6 | 5.5 | 1.2 | 2.6 |  |
| met | 10.8 | 4.8 | 3.8 | 4.6 | 0.7 | 4.6 | 4.9 | 7 | 1.7 | 4.7 | 11.4 | 5.2 | 2.8 | 3.3 | 5.1 | 7.4 | 6.3 | 0.9 | 2 | 6.8 |
| phe | 9.6 | 3.7 | 5.2 | 6.5 | 1.2 | 6.4 | 2.7 | 7.9 | 1.9 | 6.7 | 7.4 | 5 | 2.5 | 3.9 | 3.6 | 8 | 5.8 | 1.3 | 3.3 | 6.3 |
| pro | 8.4 | 3.6 | 4.6 | 5.4 | 0.7 | 7.6 | 5.2 | 5.4 | 2.3 | 6.1 | 11.2 | 5.5 | 2.4 | 4.2 | 2.8 | 6.5 | 5.4 | 1.4 | 2.9 | 7.5 |
| ser | 9.1 | 4.6 | 3.7 | 5 | 1 | 5.4 | 5.2 | 7.2 | 2.6 | 6 | 11.6 | 4.5 | 2.2 | 4.1 | 4.1 | 6.5 | 5 | 1.2 | 3.2 | 6.8 |
| thr | 9.1 | 4.2 | 3.7 | 5.6 | 0.9 | 5.7 | 5.7 | 7.5 | 2.2 | 5.5 | 12 | 4.2 | 2 | 3.5 | 5.5 | 6.2 | 5.3 | 1.1 | 2.6 | 6.7 |
| trp | 7.1 | 6.3 | 3.2 | 4.8 | 1.3 | 3.9 | 8.5 | 6.6 | 3.6 | 5 | 14.2 | 3.2 | 2.4 | 4.6 | 3.9 | 5.8 | 4.3 | 1.3 | 3 | 6.1 |
| tyr | 7.9 | 6.5 | 3.6 | 4.9 | 1.2 | 4.5 | 7 | 7.1 | 2.6 | 5 | 11.7 | 4 | 1.6 | 4.7 | 4.9 | 6.4 | 4.6 | 1.5 | 3.4 | 5.7 |
| val | 9.6 |  | 4.4 | 5.9 | 1 | 6.2 | 3. |  | 1.8 | 65 | 10.2 | 5.2 | 2.5 | 37 | 3.8 |  |  |  |  |  |

shewanella

## bovine

| Na | ala | arg |  |  |  |  |  | gly | his | ile | leu | lys |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ala | 11.4 | 5.9 | 3.1 | 4.5 | 1.9 | 5.8 | 3.6 | 7.7 | 1.9 | 4.3 | 9.7 | 4.3 | 2.1 | 3.7 | 6.4 | 6.4 | 5.6 | 1.1 | 2.6 | . 8 |
| arg | 8.5 | 7.7 | 4 | 4.6 | 2.3 | 5.9 | 3.8 | 7.6 | 2.5 | 4.4 | 9.2 | 5 | 1.7 | 4 | 5.3 | 6.3 | 5 | 1.5 | 3.4 | 6.5 |
| asn | 6.3 | 4.9 | 4.9 | 4.4 | 2.1 | 5.3 | 4.1 | 6.9 | 2.2 | 5.6 | 9.7 | 5.4 | 2.1 | 4.1 | 5.9 | 7.3 | 5.3 | 1.9 | 4.6 | 6.2 |
| asp | 7.4 | 4.9 | 3.5 | 5.4 | 2.4 | 6.6 | 3.4 | 7.4 | 2.1 | 5.4 | 9.5 | 4.7 | 2 | 4.4 | 5.4 | 6.8 | 5.7 | 1.6 | 4 | . 4 |
| cys | 6.9 | 5.9 | 4 | 5.4 | 2.7 | 5.6 | 4.9 | 7.1 | 3 | 4.4 | 8.8 | 5.4 | 1.6 | 3.5 | 6.8 | . 4 | 5.7 | . 4 | 2.7 | 5.7 |
| glu | 7.8 | 5.3 | 4.3 | 6.4 | 1.9 | 9.7 | 3.7 | 6.8 | 2 | 5.1 | 8.2 | 6.2 | 2.2 | 3.3 | 4.8 | 5.3 | 5.4 | 1.2 | 3.2 | 6.2 |
| g | 7.9 | 5.6 | 4.2 | 5 | 2 | 6.6 | 5.1 | 6.9 | 2.1 | 4.7 | 9.3 | 5.7 | 2 | 3.3 | 5.9 | 5.7 | 6.1 | 1.6 | 3.3 | . 2 |
| gly | 7.9 | 5.8 | 3.9 | 5 | 1.9 | 6.2 | 3.5 | 8 | 1.8 | 4.7 | 8.7 | 5.2 | 1.7 | 3.7 | 6.9 | 7.4 | 5.8 | 1.4 | . 2 | 2 |
| his | 6 | 5.8 | 4.3 | 3.5 | 2.9 | 5.1 | 4.1 | 6.3 | 3.2 | 4.5 | 10.6 | 4.8 | 1.6 | 4.5 | 6.7 | 6.6 | 6.1 | 1.7 | . 9 | 6 |
| ile | 6.2 | 4.9 | 4.9 | 4.7 | 2.4 | 5.3 | 4.6 | 5.8 | 2.2 | 6 | 9.9 | 5.3 | 2.1 | 4.1 | 5.3 | 7.7 | 6.9 | 1.2 | 3.7 | 6 |
| leu | 7.7 | 5.6 | 4.1 | 4.7 | 2.1 | 5.8 | 4.5 | 6.8 | 2. | 4.6 | 11 | 5. | 1.9 | 3.7 | 5.7 | 7 | 5.5 | 1.2 | . 1 | 6.4 |
| lys | 6.3 | 5.2 | 4.8 | 5.2 | 2.1 | 7.2 | 3.7 | 6.7 | 2.2 | 6 | 8.5 | 7.5 | 2 | 3.5 | 4.8 | 6.1 | 5.8 | 1.6 | 3.5 | . 3 |
| met | 9.3 | 5.3 | 4.1 | 5.9 | 1.6 | 6.1 | 3.5 | 6.4 | 1.6 | 4.1 | 9.6 | 6.6 | 2.6 | 4 | 5.1 | 6.9 | 5.5 | 1 | 3.2 | 6.6 |
| ph | 6 | 5.4 | 4.5 | 5.2 | 2.5 | 5.5 | 4.1 | 6.5 | 2.3 | 5.3 | 10.2 | 5.2 | 1.8 | 4.1 | 5.3 | 7.8 | 5.8 | 1.4 | 3.9 | 6.2 |
| pro | 8.5 | 5.4 | 3.1 | 5.1 | 1.9 | 6.7 | 3.9 | 9.5 | 1.9 | 4.3 | 7.7 | 4.3 | 1.7 | 3.3 | 8.7 | 6.9 | 5.7 | 1.4 | 2.8 | 6.4 |
| ser | 6.7 | 5.4 | 3.8 | 4.9 | 2.3 | 5.4 | 4 | 7.9 | 2.1 | 4.5 | 9.5 | 5.2 | 1.8 | 4 | 5.7 | 8.6 | 6.2 | 1.4 | 3 | 6.4 |
| thr | 7.5 | 4.6 | 3.7 | 5 | 2.6 | 5.7 | 3.8 | 6.8 | 2 | 5.2 | 9.7 | 4.4 | 1.8 | 3.9 | 6 | 7.2 | 7.3 | 1.5 | 3.5 | 6.9 |
| trp | 7.1 | 5.2 | 4.9 | 5.5 | 2.3 | 5.4 | 4.3 | 5.8 | 2.2 | 5.6 | 9.5 | 6.6 | 2.1 | 3.8 | 4.1 | 6.4 | 5.9 | 1.7 | 3.7 | 6.8 |
| tyr | 5.8 | 5.7 | 5 | 5.1 | 2.3 | 5.7 | 4.1 | 6.2 | 2.4 | 5 | 8.6 | 5.6 | 1.9 | 5 | 4.8 | 6.7 | 6.3 | 1.5 | 4.8 | 6.5 |
| val | 7.6 | 5 | 4.4 | 5.2 | 2.4 | 5.7 | 3.7 | 6.3 | 1.9 | 5 | 9.3 | 5.1 | 2.1 | 4.1 | 5.5 | 6.9 | 6.6 | 1.1 | 3.6 | 7.4 |

## Coding Signal Detection (5)

- in-frame versus any-frame dicodons


In-frame:
ATG TTG
GAT GCC
CAG AAG

Not in-frame:
TGTTGG, ATGCCC
AGAAG ., GTTGGA
AGCCCA, AGAAG ..
more
sensitive


## Computational Model (1)

- Preference model:
$K$ for each dicodon $X$ (e.g., AAA AAA), calculate its frequencies in coding and non-coding regions, $\mathrm{FC}(\mathrm{X}), \mathrm{FN}(\mathrm{X})$
$K$ calculate $X$ 's preference value $P(X)=\log (F C(X) / F N(X))$
- Properties:
$K P(X)$ is 0 if $X$ has the same frequencies in coding and non-coding regions
$K P(X)$ has positive score if $X$ has higher frequency in coding than in non-coding region; the larger the difference the more positive the score is
$K P(X)$ has negative score if $X$ has higher frequency in non-coding than in coding region; the larger the difference the more negative the score is


## Computational Model (2)

- Example

```
AAA ATT, AAA GAC, AAA TAG have the following frequencies
FC(AAA ATT) = 1.4%, FN(AAA ATT ) = 5.2%
FC(AAA GAC) = 1.9%, FN(AAA GAC) = 4.8%
FC(AAA TAG) = 0.0%, FN(AAA TAG) = 6.3%
We have
P(AAA ATT ) = log (1.4/5.2) = -0.57
P(AAA GAC) = log (1.9/4.8) = -0.40
P(AAA TAG) = - infinity (treating STOP codons differently)
A region consisting of only these dicodons is probably a non-coding region
```

- Coding preference of a region (an any-frame model)

Calculate the preference scores of all dicodons of the region and sum them up; If the total score is positive, predict the region to be a coding region; otherwise a non-coding region.

## Computational Model (3)

- In-frame preference model (most commonly used in prediction programs)

```
Data collection step:
For each known coding region,
calculate in-frame preference score, P}\mp@subsup{P}{0}{}(X)\mathrm{ , of each dicodon X; e.g., \(\underbrace{\text { ATG TGG }} \underbrace{\text { GGC GCJ }} \ldots .\).
calculate (in-frame +1) preference score, \(P_{1}(X)\), of each dicodon \(X\); e.g., AT \(\underbrace{G}\) TGC GGC \(\underbrace{G C T} . .\).
calculate (in-frame +2 ) preference score, \(P_{2}(X)\), of each dicodon \(X\); e.g., ATG \(\underbrace{\text { TGC CGC }} \underbrace{\text { GCT ..... }}\).
```


## Application step:

For each possible reading frame of a region, calculate the total in-frame preference score $\Sigma P_{0}(X)$, the total (in-frame +1 ) preference score $\Sigma P_{1}(X)$, the total (in-frame +2 ) preference score $\Sigma P_{2}(X)$, and sum them up If the score is positive, predict it to be a coding region; otherwise non-coding

## Computational Model (4)

- Prediction procedure of coding region

Procedure:
Calculate all ORFs of a DNA segment;
For each ORF, do the following
slide through the ORF with an increment of 10 base-pairs calculate the preference score, in same frame of ORF, within a window of 60 base-pairs; and assign the score to the center of the window

Example (forward strand in one particular frame)


## Computational Model (5)

- Making the call: coding or non-coding and where the boundaries are

- Need a training set with known coding and non-coding regions
$K$ select threshold(s) to include as many known coding regions as possible, and in the same time to exclude as many known non-coding regions as possible

If threshold $=0.2$, we will include $90 \%$ of coding regions and also $10 \%$ of non-coding regions If threshold $=0.4$, we will include $70 \%$ of coding regions and also $6 \%$ of non-coding regions If threshold $=0.5$, we will include $60 \%$ of coding regions and also $2 \%$ of non-coding regions

## Computational Model (6)

- Why dicodon (6mer)?

Codon (3mer) -based models are not nearly as information rich as dicodon-based models
Tricodon (9mers)-based models need too many data points for it to be practical
People have used 7-mer or 8-mer based models; they could provide better prediction methods 6-mer based models

```
There are
\[
4 * 4 * 4=64 \text { codons }
\]
\[
4^{*} 4^{*} 4^{*} 4^{*} 4^{*} 4=4,096 \text { di-codons }
\]
\[
4^{*} 4^{*} 4^{*} 4^{*} 4^{*} 4^{*} 4^{*} 4^{*} 4=262,144 \text { tricodons }
\]
```

To make our statistics reliable, we would need at least ~15 occurrences of each X-mer; so for tricodon-based models, we need at least $15^{*} 262144=3932160$ coding bases in our training data, which is probably not going to be available for most of the genomes

## Lecture Outline

- Protein-encoding genes and gene structures
- Computational models for coding regions
- Computational models for coding-region boundaries
- Markov chain model for coding regions

Signals for
Coding-Region Boundaries (1)

- Possible boundaries of an exon

- Both splice junction sites and translation starts have certain distribution profiles
- Acceptor site (human genome)
$K$ if we align all known acceptor sites (with their splice junction site aligned), we have the following nucleotide distribution

| $\mathbf{Y}_{75} \mathbf{Y}_{72} \mathbf{Y}_{78} \mathbf{Y}_{79} \mathbf{Y}_{77} \mathbf{Y}_{80} \mathbf{Y}_{66} \mathbf{Y}_{78} \mathbf{Y}_{85} \mathbf{Y}_{84} \mathbf{N} \mathbf{C}_{68} \mathbf{A} \mathbf{G}^{\prime} \mathbf{G}_{63}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -14 | -13 | -12 | -11 | -10 | -9 | -8 | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 1 |
| A | 11.1 | 12.7 | 3.2 | 4.8 | 12.7 | 8.7 | 16.7 | 16.7 | 12.7 | 9.5 | 26.2 | 6.3 | 100 | 0.0 | 21.4 |
| C | 36.5 | 30.9 | 19.1 | 23.0 | 34.9 | 39.7 | 34.9 | 40.5 | 40.5 | 36.5 | 33.3 | 68.2 | 0.0 | 0.0 | 7.9 |
| G | 9.5 | 10.3 | 15.1 | 12.7 | 8.7 | 9.5 | 16.7 | 4.8 | 2.4 | 6.3 | 13.5 | 0.0 | 0.0 | 100 | 62.7 |
| U | 38.9 | 41.3 | 58.7 | 55.6 | 42.1 | 40.5 | 30.9 | 37.3 | 44.4 | 47.6 | 27.0 | 25.4 | 0.0 | 0.0 | 7.9 |

- Donor site (human genome)

|  | -3 | -2 | -1 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{A}$ | 34.0 | 60.4 | 9.2 | 0.0 | 0.0 | 52.6 | 71.3 | 7.1 | 16.0 |
| $\mathbf{C}$ | 36.3 | 12.9 | 3.3 | 0.0 | 0.0 | 2.8 | 7.6 | 5.5 | 16.5 |
| $\mathbf{G}$ | 18.3 | 12.5 | 80.3 | 100 | 0.0 | 41.9 | 11.8 | 81.4 | 20.9 |
| $\mathbf{U}$ | 11.4 | 14.2 | 7.3 | 0.0 | 100 | 2.5 | 9.3 | 5.9 | 46.2 |

## Model for Splice Sites (1)

- Information content
$K$ for a weight matrix, the information content of each column is calculated as

$$
-F(A)^{*} \log (F(A) / .25)-F(C)^{*} \log (F(C) / .25)-F(G) * \log (F(G) / .25)-F(T) * \log (F(T) / .25)
$$

$K$ when a column has evenly distributed nucleotides, the information content is lowest

|  | -3 | -2 | -1 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{A}$ | 34.0 | 60.4 | 9.2 | 0.0 | 0.0 | 52.6 | 71.3 | 7.1 | 16.0 |
| $\mathbf{C}$ | 36.3 | 12.9 | 3.3 | 0.0 | 0.0 | 2.8 | 7.6 | 5.5 | 16.5 |
| $\mathbf{G}$ | 18.3 | 12.5 | 80.3 | 100 | 0.0 | 41.9 | 11.8 | 81.4 | 20.9 |
| $\mathbf{U}$ | 11.4 | 14.2 | 7.3 | 0.0 | 100 | 2.5 | 9.3 | 5.9 | 46.2 |

$$
\begin{aligned}
& \text { column }-3:-0.34^{*} \log (.34 / .25)-0.363^{*} \log (.363 / .25)-0.183^{*} \log (.183 / .25)- \\
& 0.114^{*} \log (.114 / .25)=0.04 \\
& \text { column }-1:-0.092^{*} \log (.092 / .25)-0.03^{*} \log (.033 / .25)-0.803^{\star} \log (.803 / .25)- \\
& 0.073^{*} \log (.073 / .25)=0.30
\end{aligned}
$$

## Model for Splice Sites (2)

- Weight matrix model
$K$ build a weight matrix for donor, acceptor, translation start site, respectively
$k$ using positions with high information
- Application of weight matrix model

|  | -3 | -2 | -1 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{A}$ | 34.0 | 60.4 | 9.2 | 0.0 | 0.0 | 52.6 | 71.3 | 7.1 | 16.0 |
| $\mathbf{C}$ | 36.3 | 12.9 | 3.3 | 0.0 | 0.0 | 2.8 | 7.6 | 5.5 | 16.5 |
| $\mathbf{G}$ | 18.3 | 12.5 | 80.3 | 100 | 0.0 | 41.9 | 11.8 | 81.4 | 20.9 |
| $\mathbf{U}$ | 11.4 | 14.2 | 7.3 | 0.0 | 100 | 2.5 | 9.3 | 5.9 | 46.2 |

$K$ add up frequencies of corresponding letter in corresponding positions

$$
\begin{aligned}
& \text { AAGGTAAGT: } 0.34+0.60+0.80+1.0+1.0+0.52+0.71+0.81+0.46=6.24 \\
& \text { TGTGTCTCA: } 0.11+0.12+0.03+1.0+1.0+0.02+0.07+0.05+0.16=2.56
\end{aligned}
$$

## Lecture Outline

- Protein-encoding genes and gene structures
- Computational models for coding regions
- Computational models for coding-region boundaries
- Markov chain model for coding regions


## Why Markov Chain?

- Preference model cannot capture all the dependence relationship among adjacent dicodons
- Markov chain model has been a popular model for modeling dependence in a linear sequence (a chain of events)
- Basic assumption of the model for a chain of events:

The "occurrence" of each event depends only on the most recent events right before this event

- Example: the weather of today is a function of only the weather of past seven days (i.e., it is independent of the weather of eight days ago)


## Markov Chain Model (1)

- Basics of probabilities:
$K P(A)$ represents the probability of $A$ being true
$K P(A, B)$ represents the event of having both $A$ and $B$ being true
$K$ if $A$ and $B$ are independent, $P(A, B)=P(A){ }^{*} P(B)$
$K P(A \mid B)$, conditional probability, of $A$ being true under the condition $B$ is true (this applies only when $B$ is true)
- Zero-th order Markov chain is equivalent to "all events are independent"
- First order Markov chain: the occurrence of an event depends only on the event right before it

$$
\mathrm{P}\left(\mathrm{~A}_{1} \mathrm{~A}_{2} \mathrm{~A}_{3} \mathrm{~A}_{4} \mathrm{~A}_{5} \mathrm{~A}_{6}\right)=\mathrm{P}\left(\mathrm{~A}_{1}\right) \mathrm{P}\left(\mathrm{~A}_{2} \mid \mathrm{A}_{1}\right) \mathrm{P}\left(\mathrm{~A}_{3} \mid \mathrm{A}_{2}\right) \mathrm{P}\left(\mathrm{~A}_{4} \mid \mathrm{A}_{3}\right) \mathrm{P}\left(\mathrm{~A}_{5} \mid \mathrm{A}_{4}\right) \mathrm{P}\left(\mathrm{~A}_{6} \mid \mathrm{A}_{5}\right)
$$

## Markov Chain Model (2)

- K-th order Markov chain model:

$$
\begin{aligned}
& \text { Example of } 5^{\text {th }} \text { order Markov chain: } \\
& P\left(A_{1} A_{2} A_{3} A_{4} A_{5} A_{6} A_{7} A_{8} A_{9} A_{10} A_{11}\right)=P\left(A_{1} A_{2} A_{3} A_{4} A_{5}\right)^{*} \\
& P\left(A_{6} \mid A_{1} A_{2} A_{3} A_{4} A_{5}\right)^{*} P\left(A_{7} \mid A_{2} A_{3} A_{4} A_{5} A_{6}\right)^{*} \\
& P\left(A_{8} \mid A_{3} A_{4} A_{5} A_{6} A_{7}\right)^{*} P\left(A_{9} \mid A_{4} A_{5} A_{6} A_{7} A_{8}\right)^{*} \\
& P\left(A_{10} \mid A_{5} A_{6} A_{7} A_{8} A_{9}\right)^{*} P\left(A_{11} \mid A_{6} A_{7} A_{8} A_{9} A_{10}\right) \\
& \hline
\end{aligned}
$$

- Markov chain model allows us to "decompose" a large problem into a collection of smaller problems


## Markov Chain Model (3)

- Definition of conditional probability

$$
P(A \mid B)=P(A, B) / P(B)
$$

- Decomposition rule


Markov Chain Model for Coding Region (1)

- Any-frame Markov chain model

Bayesian formula for coding:

$$
\begin{aligned}
& P\left(\text { coding } \mid A_{1} \ldots A_{n}\right)=P\left(\operatorname{coding}, A_{1} \ldots A_{n}\right) / P\left(A_{1} \ldots A_{n}\right) \\
& P\left(A_{1} \ldots A_{n} \mid \text { coding }\right) * P \text { (coding) }
\end{aligned}
$$

$$
\begin{aligned}
& P\left(A_{1} \ldots A_{n} \mid \text { coding }\right) P(\text { coding })+P\left(A_{1} \ldots A_{n} \mid \text { noncoding }\right) P(\text { noncoding })
\end{aligned}
$$

Bayesian formula for non-coding:

$$
\begin{aligned}
& \left.P \text { (non-coding } \mid A_{1} \ldots A_{n}\right)
\end{aligned}
$$

## Markov Chain Model for Coding Region (2)

This formula decomposes a problem of "predicting a region $A_{1} \ldots . . . A_{n}$ being a (non) coding region" to the following four problems

1. Estimating probability of seeing $A_{1} \ldots A_{n}$ in noncoding regions
2. Estimating probability of coding bases in a whole genome
3. Estimating probability of noncoding bases in a whole genome
4. Estimating probability of seeing $A_{1} \ldots A_{n}$ in coding regions

All these can be estimated using known coding and noncoding sequence data

## Markov Chain Model for Coding Region (3)

- Any-frame Markov chain model

Markov chain model (5 ${ }^{\text {th }}$ order) :

$P\left(A_{1} \ldots A_{n} \mid\right.$ coding $)=P\left(A_{1} A_{2} A_{3} A_{4} A_{5} \mid \text { coding }\right)^{*} P\left(A_{6} \mid A_{1} A_{2} A_{3} A_{4} A_{5}\right.$, coding $)$ *
$P\left(A_{7} \mid A_{2} A_{3} A_{4} A_{5} A_{6} \text {, coding }\right)^{*} \ldots{ }^{*} P\left(A_{n} \mid A_{n-5} A_{n-4} A_{n-3} A_{n-2} A_{n-1}\right.$, coding $)$
Markov chain model ( $5^{\text {th }}$ order) :
$P\left(A_{1} \ldots A_{n} \mid\right.$ noncoding $)=P\left(A_{1} A_{2} A_{3} A_{4} A_{5} \mid\right.$ noncoding $) * P\left(A_{6} \mid A_{1} A_{2} A_{3} A_{4} A_{5}\right.$, noncoding $) *$ $P\left(A_{7} \mid A_{2} A_{3} A_{4} A_{5} A_{6} \text {, noncoding }\right)^{*} \ldots .{ }^{*} P\left(A_{n} \mid A_{n-5} A_{n-4} A_{n-3} A_{n-2} A_{n-1}\right.$, noncoding $)$

## P(coding): total \# coding bases/total \# all bases

P(noncoding): total \# noncoding bases/total \# all bases

## Build Markov Tables (1)

- a priori probability tables ( $5^{\text {th }}$ order): P (5mer |coding) and P (5mer | noncoding)
$\measuredangle 5$ mer frequency table for coding regions
$K 5$ mer frequency table for noncoding regions
- Conditional probability tables (5 $5^{\text {th }}$ order): $\mathrm{P}(\mathrm{X} \mid 5 \mathrm{mer}$, coding) and $\mathrm{P}(\mathrm{X} \mid 5 \mathrm{mer}$, noncoding) ( X could be $\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}$ )
$\measuredangle$ For a fixed 5 mer (e.g., ATT GT), what is the probability to have A, C, G or T following it in coding region
$\measuredangle$ For a fixed 5 mer (e.g., ATT GT), what is the probability to have A, C, G or T following it in noncoding region
- $\mathrm{P}($ coding $)=\sim 0.02$ and $\mathrm{P}($ noncoding $)=\sim 0.98$


## Build Markov Tables (2)

| a priori probabilities <br> for coding PAC |
| :--- |
| AAA AA: 0.000012 |
| AAA AC: 0.000001 |
| AAA AG: 0.000101 |
| $\ldots \ldots$. |

a priori probabilities
for noncoding PAN
AAA AA: 0.000329
AAA AC: 0.000201
AAA AG: 0.000982
$\ldots . . .$.

AAAAA: 0.000329
AAA AC: 0.000201
AAA AG: 0.000982
conditional probabilities for coding PC
conditional probabilities for noncoding PN

## A C G T

AAA AA: 0.170 .390 .010 .43
AAA AC: 0.120 .440 .020 .42
AAA AG: 0.010 .690 .100 .20

A C G T
AAA AA: 0.710 .090 .000 .20
AAA AC: 0.610 .190 .020 .18
AAA AG: 0.010 .690 .100 .20

## In-Frame

## Markov Chain Model (1)

- In-frame Markov tables
a priori probabilities for coding $\mathrm{PACO}_{0,1,2}$

| AAA AA: 0.000012 | 0.000230 | 0.000009 |
| :--- | :--- | :--- |
| AAA AC: 0.000001 | 0.000182 | 0.000011 |
| AAA AG: 0.000101 | 0.000301 | 0.000101 |
| $\ldots \ldots .$. |  |  |

conditional probabilities for coding $\mathrm{PC}_{0}$

AAA AA: 0.170 .390 .010 .43
AAA AC: 0.120 .440 .020 .42
AAA AG: 0.010 .690 .100 .20
conditional probabilities for coding $\mathrm{PC}_{1}$

AAA AA: 0.330 .120 .100 .35
AAA AC: 0.020 .490 .120 .37
AAA AG: 0.100 .600 .150 .15
translation frame

conditional probabilities for coding $\mathrm{PC}_{2}$

AAA AA: 0.170 .390 .010 .43
AAA AC: 0.120 .440 .020 .42
AAA AG: 0.010 .690 .100 .20

## In-Frame

## Markov Chain Model (2)

- In-frame Markov chain (5th order) calculation

$$
\begin{aligned}
& P_{0}\left(A_{1} \ldots A_{n} \mid \text { coding }\right)=P_{0}\left(A_{1} A_{2} A_{3} A_{4} A_{5} \mid \text { coding }\right)^{*} P_{0}\left(A_{6} \mid A_{1} A_{2} A_{3} A_{4} A_{5} \text {, coding }\right)^{*} \\
& P_{1}\left(A_{7} \mid A_{2} A_{3} A_{4} A_{5} A_{6} \text {, coding }\right)^{*} P_{2}\left(A_{8} \mid A_{3} A_{4} A_{5} A_{6} A_{7} \text {, coding }\right)^{*} \ldots \ldots . .
\end{aligned}
$$

\section*{| 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | A A A A C T G C .......}

$$
\begin{aligned}
& P\left(A_{1} \ldots A_{n} \mid \text { noncoding }\right)=P\left(A_{1} A_{2} A_{3} A_{4} A_{5} \mid \text { noncoding }\right){ }^{*} P\left(A_{6} \mid A_{1} A_{2} A_{3} A_{4} A_{5} \text {, noncoding }\right) * \\
& P\left(A_{7} \mid A_{2} A_{3} A_{4} A_{5} A_{6} \text {, noncoding }\right)^{*} \ldots .{ }^{*} P\left(A_{n} \mid A_{n-5} A_{n-4} A_{n-3} A_{n-2} A_{n-1} \text {, noncoding }\right)
\end{aligned}
$$

Calculation for non-coding regions stays the same

- Markov tables for Human genome



## in-frame conditional probabilities

■. 1867120.1701700 .4273900 .215729 $0.259907 \quad 0.3386520 .1101390 .291302$ $0.2824350 .2290420 .305389 \quad 0.183134$ 0.2289370 .2228350 .2710630 .277165 $\begin{array}{lll}0.260952 & 0.246270 & 0.384087 \\ 0.108691\end{array}$ 0.2952710 .3428750 .0476040 .314250 $0.214186 \quad 0.3285110 .2286510 .228651$ $0.145116 \quad 0.2473650 .430077 \quad 0.177442$ $0.301228 \quad 0.2112050 .285700 \quad 0.201867$ 0.2914710 .3497950 .0762020 .282532 $0.374479 \quad 0.197526 \quad 0.251042 \quad 0.176953$ $0.124338 \quad 0.302720 \quad 0.410809 \quad 0.162134$ 0.0000010 .5898690 .0000010 .410131 0.2263340 .3490590 .0755470 .349059 $0.0000010 .364719 \quad 0.270561 \quad 0.364719$ $0.153399 \quad 0.460197 \quad 0.1165030 .269902$ 0.1667200 .2908260 .3725300 .169924 $0.324470 \quad 0.4042410 .085158 \quad 0.186131$ $\begin{array}{llll}0.164926 & 0.448450 & 0.185647 & 0.200977\end{array}$ $\begin{array}{llll}0.050185 & 0.414194 & 0.343088 & 0.192534\end{array}$ 0.1983590 .1619690 .5425430 .097128 $0.2378350 .422977 \quad 0.1056720 .233516$ $\begin{array}{lll}0.111173 & 0.512849 & 0.273464 \\ 0.102514\end{array}$ $\begin{array}{llll}0.059626 & 0.291251 & 0.540305 & 0.108818\end{array}$ $0.048228 \quad 0.3553150 .469882 \quad 0.126575$ 0.2046050 .4318600 .2272560 .136279 $\begin{array}{llll}0.094442 & 0.378024 & 0.417396 & 0.110139\end{array}$ 0.0365820 .2569720 .5964020 .110045 0.0000010 .7372940 .0000010 .262706

## In-Frame

## Markov Chain Model (4)

- Coding score procedure
$K$ for a DNA segment [i, j], calculate Markov coding scores scoreC[0], scoreC[1], scoreC[2], representing three frames (one strand), and noncoding score scoreN
$K$ if MAX \{ scoreC[0], scoreC[1], scoreC[2] \} > scoreN, the region is predicted to coding; otherwise non-coding

calculated reading frame in reference to the starting point of the first base


## Application of Markov Chain Model

- Prediction procedure

> Procedure:
> Calculate all ORFs of a DNA segment;
> For each ORF, do the following
> slide through the ORF with an increment of 10 base-pairs
> calculate the preference score, in same frame of ORF, within a window of 60 base-pairs; and assign the score to the center of the window

- A computing issue

Multiplication of many small numbers (probabilities) is generally problematic in computer

Converting a * b * c * $\mathrm{d} \ldots . .{ }^{*} \mathrm{z}$ to $\log (\mathrm{a})+\log (\mathrm{b})+\log (\mathrm{c})+\log (\mathrm{d}) \ldots \log (\mathrm{z})$

## References

- Chapter 9 in "Current Topics in Computational Molecular Biology, edited by Tao Jiang, Ying Xu, and Michael Zhang. MIT Press. 2002."
- Chapter 9 in "Pavel Pevzner: Computational Molecular Biology An Algorithmic Approach. MIT Press, 2000."


## Selected Reading

- http://www.ncbi.nlm.nih.gov/pubmed/20221925
- http://www.ncbi.nlm.nih.gov/pubmed/12364589
- http://www.ncbi.nlm.nih.gov/pubmed/16728949
- http://www.ncbi.nlm.nih.gov/pubmed/21653517
- http://www.ncbi.nIm.nih.gov/pubmed/19564452
- http://www.ncbi.nIm.nih.gov/pubmed/19494180
- http://www.ncbi.nlm.nih.gov/pubmed/10779491


## Acknowledgments

This file is for the educational purpose only. Some materials (including pictures and text) were taken from the Internet at the public domain.

